Problem 1. (10 Points)

a. Is the proposition

   If $1 < 0$, then $3 = 4$.

   True or false? Why ?

b. Find a proposition with two variables p and q that is **never true**. Don't prove your answer.

c. Write a proposition equivalent to $p \vee \neg q$ that uses only $p,q,\neg$ and the connective $\wedge$. Don't prove your answer.

Problem 2. (10 Points)

a. Prove that the proposition "if it is not hot, then it is hot" is equivalent to "it is hot".
   Hint: Let p denote the proposition "it is hot"

b. Determine whether the proposition $((p \rightarrow \neg q) \wedge q) \rightarrow \neg p$ is a tautology:

Problem 3. (10 Points)

In this problem, suppose the variable x represents students and y represents courses, and consider the predicates:

$M(y)$: y is a math course $S(x)$: x is a sophomore, $F(x)$: x is a full-time student $T(x,y)$: x is taking y.

Consider the following English statements:

1. Every student is taking a course
2. Some student is taking every course
3. Every full-time sophomore is taking a math course
4. Some full-time sophomore is taking a math course

What does each of the following represent (Circle one number).

a. $\exists x \forall y T(x,y)$ 1   2   3   4

b. $\forall x \exists y[(B(x) \wedge F(x)) \rightarrow (M(y) \wedge T(x,y))]$.   1   2   3   4

Problem 4.(10 Points)

In this problem, suppose the variable x represents students and y represents courses, and consider the predicates:

$M(y)$: y is a math course $S(x)$: x is a sophomore, $F(x)$: x is a full-time student $T(x,y)$: x is taking y.

Write the statements below using these predicates and any needed quantifiers.

a. Some students are not sophomore.

b. Every sophomore is a full-time student and is taking a math course

Problem 5.(10 Points)

a. Show that the following argument is valid:

$$\begin{array}{l} \ \ p \lor q \\ \neg p \lor r \\ \hline \therefore\ q \lor r \end{array}$$

b. Use (a) to show that the hypotheses “I left my notes in the library or I finished the rough draft of the paper” and “I did not leave my notes in the library or I revised the bibliography” imply that “I finished the rough draft of the paper or I revised the bibliography”.

Problem 6. (5 Points)

Show that the following argument is valid:

She is a Math Major or a Computer Science Major.
If she does not know discrete math, she is not a Math Major.
If she knows discrete math, she is smart.
She is not a Computer Science Major.
Therefore, she is smart.

Hint: Use the symbols *m*, *c*, *d*, *s*, to represent the propositions She is a Math Major, She is a Computer Science Major, She knows discrete math, she is smart respectively.

Problem 7. (10 Points)

Suppose $B = \{x, \{x\}\}$. Mark the statement as TRUE or FALSE (Circle the right answer)

a. $\{x\} \in B$. TRUE FALSE

b. $\{x\} \subseteq B$. TRUE FALSE

c. $x \subseteq B$. TRUE FALSE

d. $\varnothing \in P(B)$. TRUE FALSE

e. $|P(B)| = 4$ TRUE FALSE

Problem 8.(10 Points)
Prove the following:

a. $\overline{A \cup \overline{B}} \cup \overline{A} = \overline{A}$

b.If $A \cap B = A \cup B$, then $A = B$.

Problem 9 (10 Points)

Consider the function:

$$f : \mathbf{Z} \to \mathbf{Z} \text{ where } f(x) = \begin{cases} x - 2 & \text{if } x \geq 5 \\ x + 1 & \text{if } x \leq 4. \end{cases}$$

a. Is f one-to-one? Why?

b. Is f onto? Why ?

Problem 10 (10 Points).

a. Suppose $g : A \to B$ and $f : B \to C$, where $f \circ g$ is one-to-one and f is one-to-one. Show that g is one-to-on.

b. Suppose $g : A \to B$ and $f : B \to C$, where $f \circ g$ is one-to-one and g is one-to-one. Must f be 1-1? Why?

Problem 11.(5 Points)

Suppose $f : \mathbf{R} \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ where $g(x) = 2x + 1$ and $g \circ f(x) = 2x + 11$. Find the rule for f.

Problem 12 (10 Points)

For each of the following, find a formula that generates the sequence $a_1, a_2, a_3 \ldots$

a. 5,9,13,17,21,….

$a_n =$

b. 15,20,25,30,35,….

$a_n =$

c. 0,2,0,2,0,2,0,....

$a_n =$

Problem 13 (15 Points)

a. (7) Show that the set of natural numbers divisible by 5 but not by 4 is countable

b. (8) Show that the union of two countably infinite sets is countably infinite

Problem 14 (10 Points)

Suppose $g : \mathbf{R} \to \mathbf{R}$ where $g(x) = \left\lfloor \frac{x-1}{2} \right\rfloor$.

a. If $S = \{x \mid 1 \le x \le 6\}$, find $g(S)$.

b. If $T = \{2\}$, find $g^{-1}(T)$.

Problem 15 (5 Points)

Show that $\lceil x \rceil = -\lfloor -x \rfloor$.